

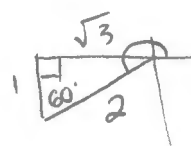
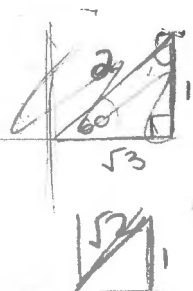
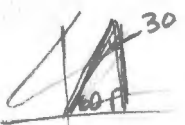
+4  
4

Use the angle sum or difference identity to find the exact value of each.

1)  $\cos \frac{7\pi}{12} = \cos 105 = \cos(150 - 45)$

2)  $\tan 255^\circ = \tan(210^\circ + 45^\circ)$   
 $\tan 255^\circ = \frac{\tan 210^\circ + \tan 45^\circ}{1 - \tan 210^\circ \cdot \tan 45^\circ}$   
 $= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1}$

$\cos(A+B) = \cos A \cdot \cos B + \sin A \cdot \sin B$   
 $= \cos 150 \cdot \cos 45 + \sin 150 \cdot \sin 45 = \frac{-\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$



$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$   
 $\frac{-\sqrt{6} + \sqrt{2}}{4}$



$\frac{A}{T/C} = \frac{-\sqrt{6} + \sqrt{2}}{4}$

100%

A+

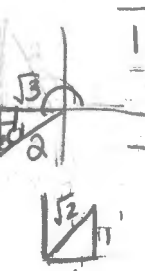
Perfect!!!



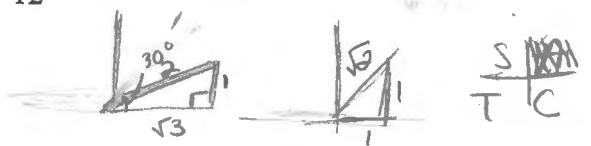
$\frac{(\frac{1}{\sqrt{3}} + 1) \cdot \sqrt{3}}{(1 - \frac{1}{\sqrt{3}}) \cdot \sqrt{3}} = \frac{(1 + \sqrt{3})(-\sqrt{3} - 1)}{(\sqrt{3} - 1)(-\sqrt{3} - 1)}$   
 $\frac{-\sqrt{3} - 1 - \sqrt{9} - \sqrt{3}}{-\sqrt{9} - \sqrt{3} + \sqrt{3} + 1}$   
 $\frac{-2\sqrt{3} - 4}{-3 + 1} = \frac{-2\sqrt{3} - 4}{-2}$   
 $= \frac{-2\sqrt{3}}{-2} + \frac{-4}{-2} = \sqrt{3} + 2$

3)  $\tan 165^\circ = \tan(210 - 45)$   
 $= \frac{\tan 210 - \tan 45}{1 + \tan 210 \cdot \tan 45}$

4)  $\tan \frac{5\pi}{12} = \tan 75^\circ (45^\circ + 30^\circ)$   
 $= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \cdot \tan 45^\circ}$



$\frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}} \cdot 1} = \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{(1 - \sqrt{3})(\sqrt{3} + 1)}{(\sqrt{3} + 1)(\sqrt{3} + 1)}$



$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1} = \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{(1 + \sqrt{3})(-\sqrt{3} - 1)}{(\sqrt{3} - 1)(-\sqrt{3} - 1)}$   
 $\frac{-\sqrt{3} - 1 - \sqrt{9} - \sqrt{3}}{-\sqrt{9} + \sqrt{3} - \sqrt{3} + 1}$   
 $\frac{-2\sqrt{3} - 4}{-2} = \frac{-2\sqrt{3}}{-2} + \frac{-4}{-2} = \sqrt{3} + 2$

$\frac{(1 - \sqrt{3})(\sqrt{3} + 1)}{(\sqrt{3} + 1)(\sqrt{3} + 1)}$   
 $= \frac{-\sqrt{3} + 1 - \sqrt{9} - \sqrt{3}}{-\sqrt{9} + \sqrt{3} - \sqrt{3} + 1}$   
 $\frac{-2\sqrt{3} + 4}{-2} = \frac{-2\sqrt{3}}{-2} + \frac{4}{-2}$   
 $= \sqrt{3} - 2$

Angle Sum or Difference Identities

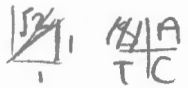
Use the angle sum or difference identity to find the exact value of each.

1)  $\sin \frac{13\pi}{12} = \sin 195^\circ$

$\sin 195^\circ = \sin(150 + 45)$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$= \sin 150 \cdot \cos 45 + \cos 150 \cdot \sin 45$   
 $= \sin \frac{1}{2} \cdot \cos \frac{1}{2} = \frac{\sqrt{3}}{2} + (-\frac{\sqrt{3}}{2}) \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}$



$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + (-\frac{\sqrt{3}}{2}) \cdot \frac{\sqrt{2}}{2}$

2)  $\sin \frac{11\pi}{12} = \sin 165^\circ = \sin(120 + 45)$

$\sin(A+B) = \sin(120 + 45)$

$= \sin A \cdot \cos B + \cos A \cdot \sin B$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + (-\frac{1}{2}) \cdot \frac{\sqrt{2}}{2}$



$\frac{\sqrt{6}}{4} + (-\frac{\sqrt{2}}{4})$



$= \frac{\sqrt{6} - \sqrt{2}}{4}$

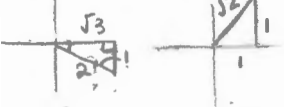
3)  $\sin -75^\circ$

$\sin -75^\circ = (-30 - 45)$

$\frac{\sqrt{2}}{4} + (-\frac{6}{4})$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$= \sin(-30) \cdot \cos 45 + \cos(-30) \cdot \sin 45$



$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - (\frac{\sqrt{3}}{2}) \cdot \frac{\sqrt{2}}{2}$

$-\frac{\sqrt{2}}{4} - (\frac{\sqrt{6}}{4}) = -\frac{\sqrt{2} + \sqrt{6}}{4}$

#1 → 5)  $\sin 195^\circ$   
look at #1

4)  $\sin \frac{5\pi}{12} = \sin 75^\circ (30 + 45)$

$= \sin A \cdot \cos B + \cos A \cdot \sin B$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$



$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$

#3 (look at #3)  
6)  $\sin -\frac{5\pi}{12} = \sin -75^\circ$

7)  $\cos \frac{7\pi}{12} = \cos 105^\circ (60 + 45)$

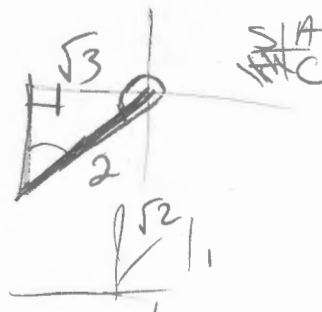
$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$



8)  $\cos 255^\circ = \cos(210^\circ + 45^\circ)$   
 $= \cos A \cdot \cos B - \sin A \cdot \sin B$



$-\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - (-\frac{1}{2}) \cdot \frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{6}}{4} - (-\frac{\sqrt{2}}{4})$

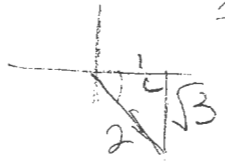
$= \frac{-\sqrt{6} + \sqrt{2}}{4}$

9)  $\sin -15^\circ (-60 + 45)$

10)  $\cos \frac{7\pi}{12}$  look at # 7

$\sin A + B = (\sin A \cdot \cos B + \cos A \cdot \sin B)$   
 $= \sin -60 \cdot \cos 45 + \cos -60 \cdot \sin 45$

$\frac{\sqrt{2}}{2} \left( -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \right)$   
 $= \frac{-\sqrt{6} + \sqrt{2}}{4}$

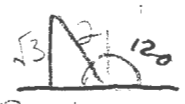


11)  $\tan 165^\circ (120 + 45)$

12)  $\tan \frac{5\pi}{12} \tan 75$

$\tan 165 = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\frac{\tan 30 + \tan 45}{1 - \tan 30 \cdot \tan 45}$



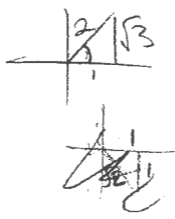
$\frac{\tan 120 + \tan 45}{1 - \tan 120 \cdot \tan 45}$   
 $= \frac{\sqrt{3} + 1}{1 - (\sqrt{3}) \cdot 1} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$   
 FOIL  
 first outside  
 Inside  
 LAST  
 $= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{1 - 3}$   
 $= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{-2}$

$\frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1} = \frac{\frac{1 + \sqrt{3}}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{(1 + \sqrt{3}) \cdot \sqrt{3}}{\sqrt{3} - 1}$   
 $= \frac{\sqrt{3} + 3}{\sqrt{3} - 1} \cdot \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = \frac{\sqrt{3} + 3}{\sqrt{3} - 1} \cdot \frac{(\sqrt{3} + 1)}{3 - 1}$   
 $= \frac{\sqrt{3} + 3}{\sqrt{3} - 1} \cdot \frac{(\sqrt{3} + 1)}{2}$

13)  $\tan \frac{7\pi}{12} \tan 105$   
 $\frac{\tan 60 + \tan 45}{1 - \tan 60 \cdot \tan 45} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

14)  $\tan \frac{\pi}{12} \tan 15 (60 - 45)$   
 $\tan A - B = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$   
 $= \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}} \cdot 1} = \frac{\frac{1 - \sqrt{3}}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{1 - \sqrt{3}}{\sqrt{3} + 1}$

$\frac{-2\sqrt{3} + 4}{-2} = \sqrt{3} - 2$



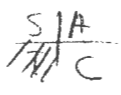
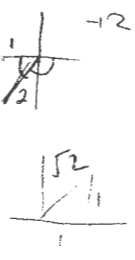
$\frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(1 - \sqrt{3})}{(1 - \sqrt{3})} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(1 - \sqrt{3})}{1 - 3}$   
 $= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(1 - \sqrt{3})}{-2}$   
 $= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{-(1 - \sqrt{3})}{2} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(\sqrt{3} - 1)}{2}$   
 $= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(\sqrt{3} - 1)}{2} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(\sqrt{3} - 1)}{2}$   
 $= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(\sqrt{3} - 1)}{2} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{(\sqrt{3} - 1)}{2}$

15)  $\tan -\frac{5\pi}{12} \tan 75 (-120 + 45)$

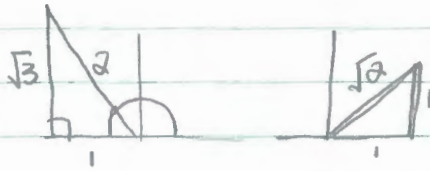
16)  $\tan 195^\circ$

$\tan A \cdot B = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \frac{\tan -120 + \tan 45}{1 - \tan -120 \cdot \tan 45}$   
 $= \frac{\sqrt{3} + 1}{1 - (\sqrt{3}) \cdot 1} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

$\frac{2\sqrt{3} + 4}{-2} = \frac{2\sqrt{3}}{-2} + \frac{4}{-2} = -\sqrt{3} - 2$



$$\textcircled{3} \tan 165^\circ = \tan(120^\circ + 45^\circ)$$



$$\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{-\frac{\sqrt{3}}{1} + \frac{1}{1}}{1 - \left(\frac{\sqrt{3}}{1}\right) \cdot \frac{1}{1}}$$

$$= \frac{-\sqrt{3} + 1}{1 + \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{-\sqrt{3} + \sqrt{9} + 1 - \sqrt{3}}{1 + \sqrt{3} + \sqrt{3} + \sqrt{9}}$$

$$\frac{-2\sqrt{3} + 4}{-2} = \frac{-2\sqrt{3}}{-2} + \frac{4}{-2}$$

$$= \sqrt{3} + (-2)$$

$$= \boxed{\sqrt{3} - 2}$$