HW #43 - Factorials & Sums

Name: Rachel Milian
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Write the first four terms of the sequence whose general term is given.

1) $a_n = \frac{2^n}{(n+2)!}$

1) D

A) $\frac{3}{2}, 1, \frac{8}{5}, \frac{3}{8}$ B) $\frac{2}{3}, 1, \frac{8}{5}, \frac{8}{3}$ C) $\frac{1}{3}, \frac{1}{6}, \frac{2}{15}, \frac{2}{45}$ D) $\frac{1}{3}, \frac{1}{6}, \frac{1}{15}, \frac{1}{45}$

100?

$a_1 = \frac{2^1}{(1+2)!} = \frac{2}{3!} = \frac{2}{3\cdot 2\cdot 1} = \frac{1}{3}$

$a_2 = \frac{2^2}{(2+2)!} = \frac{4}{4!} = \frac{4}{4\cdot 3\cdot 2\cdot 1} = \frac{1}{6}$

$a_3 = \frac{2^3}{(3+2)!} = \frac{8}{5!} = \frac{8\cdot 2}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{1}{15}$

$a_4 = \frac{2^4}{(4+2)!} = \frac{16}{6!} = \frac{16\cdot 2}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{2}{90} = \frac{1}{45}$

$a_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{15}, \frac{1}{45}$

Evaluate the factorial expression.

A+

2) $\frac{5!}{3!\,2!}$

2) D

A) 0! B) 5 C) 1 D) 10

$\frac{5!}{3!\,2!} = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(2\cdot 1)} = \frac{20}{2} = 10$

Perfect!!!

Find the indicated sum.

3) $\sum_{i=1}^{5} (i+8)$

3) C

A) 22 B) 13 C) 55 D) 42

$\sum_{i=1}^{5} (i+8) \rightarrow (1+8) + (2+8) + (3+8) + (4+8) + (5+8)$

$9 + 10 + 11 + 12 + 13$

55

4) $\sum_{i=8}^{11} \frac{1}{i+5}$

4) A

A) $\frac{6061}{21840}$ B) 58 C) $\frac{2925}{4096}$ D) $-\frac{701}{792}$

$\sum_{i=8}^{11} \frac{1}{i+5} = \frac{1}{8+5} + \frac{1}{9+5} + \frac{1}{10+5} + \frac{1}{11+5}$

$= \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}$

$\frac{A}{B} + \frac{C}{D} = \frac{A(D) + C(B)}{B\cdot D}$

$= \frac{27}{182} + \frac{31}{240} = \frac{27(240) + 31(182)}{182(240)} \rightarrow \frac{6480 + 5642}{43680}$

$\frac{12122}{43680} = \frac{6061}{21840}$

HW #43 Examples - Factorials & Sums

Name:_____

Date:_____ Per::_____

Write the first four terms of the sequence whose general term is given.

1) $a_n = \frac{2^n}{(n+2)!}$ 1) D

A) $\frac{1}{3}, \frac{1}{6}, \frac{2}{15}, \frac{2}{45}$ B) $\frac{3}{2}, 1, \frac{8}{5}, \frac{3}{8}$ C) $\frac{2}{3}, 1, \frac{8}{5}, \frac{8}{3}$ D) $\frac{1}{3}, \frac{1}{6}, \frac{1}{15}, \frac{1}{45}$

Evaluate the factorial expression.

2) $\frac{7!}{5!\,2!}$ 2) B

A) 0! B) 21 C) 7 D) 1

Find the indicated sum.

3) $\sum_{i=4}^{7} 6i$ 3) C

A) 66 B) 42 C) 132 D) 90

4) $\sum_{i=7}^{10} \frac{1}{i+6}$ 4) C

A) 58 B) $\frac{2400}{2401}$ C) $\frac{6061}{21840}$ D) $-\frac{487}{420}$

Math Analysis

Name_____

Warm-up: Sequences

Date_____ Period____

State if each sequence is arithmetic.

1) 9, 11, 14, 18, ...

a) NO

2) 8, 108, 208, 308, ...

+100 +100 +100

a) YES

3) 2, 4, 12, 48, ...

a) NO

4) 1, 9, 25, 49, ... NO arithmetic

a) NO

Find the common difference.

5) −35, −45, −55, −65, ...

$a_2 \rightarrow -45 = -35 + d(2-1)$

$-45 = -35 + d$

$-10 = d$

6) −38, −35, −32, −29, ...

$a_2 \rightarrow -35 = -38 + d(2-1)$

$-35 = -38 + d$

$3 = d$

7) 23, 15, 7, −1, ...

$a_2 \rightarrow 15 = 23 + d(2-1)$

$15 = 23 + d$

$-8 = d$

8) −36, −41, −46, −51, ...

$a_n \quad -41 = -36 + d(2-1)$

$-41 = -36 + d$

$-5 = d$

Given the explicit formula for an arithmetic sequence find the common difference.

9) $a_n = -16 - 8n$ common difference

a) -8

10) $a_n = 65 - 100n$ common difference

a) -100

Given the recursive formula for an arithmetic sequence find the common difference.

11) $a_n = a_{n-1} + 6$ → common difference
$a_1 = 2$

a) +6

12) $a_n = a_{n-1} - 200$ → common difference
$a_1 = -2$

a) -200

Find the common difference and the 52nd term.

13) −15, −8, −1, 6, ... d=7

$a_{52} \rightarrow a_1 + 7(52)$
$a_1 + 7 \cdot 52$
$a_{52} = -15 + 364$
$\boxed{a_{52} = 349}$

14) 4, 13, 22, 31, ... d=9

$a_{52} = 4 + 9(52)$
$a_{52} = 4 + 468$
$\boxed{a_{52} = 472}$

15) 3, 8, 13, 18, ... d=5

$a_{52} = 3 + 5(52)$
$a_{52} = 3 + 260$
$\boxed{a_{52} = 263}$

16) −18, −27, −36, −45, ... d=-9

$a_{52} = -18 + (-9)(52)$
$a_{52} = -18 - 468$
$\boxed{a_{52} = -468}$

Given two terms in an arithmetic sequence find the term named in the problem.

17) $a_{19} = -124$ and $a_{39} = -264$
Find a_{28}

$a_{19} \rightarrow a_1 + d(19-1) = -124$
$a_{39} \rightarrow a_1 + d(39-1) = -264$

(1) $a_1 + 18d = -124 \cdot (-1) \rightarrow -a_1 - 18d = 124$
(2) $a_1 + 38d = -264$
$20d = -140$
$d = -7$

(2) $a_1 + 38(-7) = -264$
$a_1 + -266 = -264$
$a_1 = 2$

$a_{28} \rightarrow 2 + (-7 \cdot 27) = ?$
$= 2 + (-189)$
$= \boxed{-187}$

18) $a_{15} = 104$ and $a_{36} = 293$
Find a_{30}

$a_{15} = a_1 + d(15-1) = 104$
$a_{36} = a_1 + d(36-1) = 293$

(1) $a_1 + 14d = 104 \cdot (-1) \rightarrow -a_1 - 14d = -104$
(2) $a_1 + 35d = 293$
$21d = 189$
$d = 9$

(2) $a_1 + 35(9) = 293$
$a_1 + 315 = 293$
$\boxed{a_1 = -22}$

$a_{30} = -22 + 9(30-1) = ?$
$= -22 + 261$
$= \boxed{239}$

19) $a_{17} = 140$ and $a_{40} = 370$
Find a_{34}

$a_{17} \rightarrow a_1 + 16d = 140 \cdot (-1) \rightarrow -a_1 - 16d = -140$
$a_{40} \rightarrow a_1 + 39d = 370$
$23d = 230$
$d = 10$

$a_1 + 39(10) = 370$
$a_1 + 390 = 370$
$a_1 = -20$

$-20 + 10(33) = ?$
$-20 + 330 = \boxed{310}$

20) $a_{18} = -3392$ and $a_{40} = -7792$
Find a_{37}

$a_{18} = a_1 + d(18-1) = -3392$
$a_{40} = a_1 + d(40-1) = -7792$

(1) $a_1 + 17d = -3392 \cdot (-1)$
(2) $a_1 + 39d = -7792$
$22d = -4400$
$d = -200$

$a_1 + 39(-200) = -7792$
$a_1 + (-7800) = -7792$
$a_1 = -8$

$a_{37} = -8 + 200(37-1) = ?$
$\boxed{a_{37} = -7208}$

Factorials & Sums

Name:_____

Date:_____ Per:____

Write the first four terms of the sequence whose general term is given.

1) $a_n = \frac{n^5}{(n+1)!}$ $\frac{1^5}{1+1!} = \frac{1}{2\cdot1} = \frac{1}{2}$ $\frac{2^5}{2+1!} \to \frac{32}{3\cdot2\cdot1} \to \frac{32}{4}$

1) A

A) $\frac{5}{2}, \frac{5}{3}, \frac{5}{4}, 1$ B) $\frac{1}{2}, \frac{16}{3}, \frac{81}{4}, \frac{256}{5}$ C) $\frac{1}{2}, \frac{16}{3}, \frac{81}{8}, \frac{128}{15}$ D) $\frac{5}{2}, \frac{5}{3}, \frac{5}{8}, \frac{1}{6}$

2) $a_n = \frac{(n+1)!}{n^2}$ $a_1 \to \frac{(1+1)!}{1^2} \to \frac{2!}{1} \to \frac{2\cdot1}{1} \to 1$ $a_2 \to \frac{(2+1)!}{2^2} \to \frac{3!}{4} \to \frac{3\cdot2\cdot1}{4} = \frac{6}{4} \to \frac{3}{2}$ $a_3 \to \frac{(3+1)!}{3^2} \to \frac{5\cdot4\cdot3\cdot2\cdot1}{9}$ $a_4 = \frac{5\cdot4\cdot3\cdot2\cdot1}{16} =$

2) C

A) $2, \frac{3}{2}, \frac{8}{3}, \frac{15}{2}$ B) $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$ C) $1, \frac{3}{2}, 4, 15$ D) $1, \frac{3}{2}, 2, \frac{5}{2}$

3) $a_n = 2(n+2)!$ $a_1 \to 2(1+2)! \to 2\cdot3! \to 2\cdot3\cdot2\cdot1 \to 12$ $a_2 \to 2(2+2)! \to 2\cdot4! \to 2\cdot4\cdot3\cdot2\cdot1 \to 48$ $a_3 \to 2\cdot5\cdot4\cdot3\cdot2\cdot1 \to 240$

3) B

A) 4, 12, 48, 240 B) 12, 48, 240, 1440 C) 4, 24, 144, 960 D) 12, 96, 720, 5760

4) $a_n = \frac{2n}{(n+3)!}$ $\frac{2^1}{1+3!} \to \frac{2}{4\cdot3\cdot2\cdot1} \to \frac{2}{24} \div 2 \to \frac{1}{12}$ $\frac{2^2}{2+3!} \to \frac{4}{4!} \to \frac{4}{4\cdot3\cdot2\cdot1} \to \frac{4}{24} \div 4 \to \frac{1}{6}$ $a_3 \to \frac{2^3}{3+3!} = \frac{8}{6!} = \frac{1}{90}$

4) B

A) $2, \frac{4}{5}, \frac{4}{3}, \frac{7}{16}$ B) $\frac{1}{12}, \frac{1}{30}, \frac{1}{90}, \frac{1}{315}$ C) $\frac{1}{12}, \frac{1}{30}, \frac{1}{45}, \frac{2}{315}$ D) $\frac{2}{7}, \frac{1}{2}, \frac{8}{9}, \frac{8}{5}$

5) $a_n = \frac{4(n+1)!}{n!}$ $\frac{4(1+1)!}{1!} \to \frac{4\cdot2!}{1!} \to \frac{8!}{1!}$ $\frac{4(2+1)!}{2!} \to 12$

5) B

A) $8, 6, \frac{8}{3}, \frac{5}{6}$ B) $8, 6, \frac{16}{3}, 5$ C) 5, 6, 7, 8 D) 8, 12, 16, 20

Evaluate the factorial expression.

6) $\frac{7!}{5!} \to \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1} = 42$

6) A

A) 42 B) 2! C) 7 D) $\frac{7}{5}$

7) $\frac{4!}{6!} \to \frac{4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = 30$

7) A

A) 30 B) $\frac{1}{2!}$ C) $\frac{1}{30}$ D) 2!

8) $\frac{8!}{6!\,2!} \to \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3\cdot2\cdot1\,(2\cdot1)} = \frac{56}{2} = 28$

8) A

A) 28 B) 1 C) 0! D) 8

Factorials & Sums

Find the indicated sum.

9) $\sum_{i=2}^{5} (4i - 5)$ → $(4(2)-5)+(4(3)-5)+(4(4)-5)+(4(5)-5)$ → $3+7+11+15$ → 36 — 9) C

A) 30 B) 33 C) 36 D) 23

10) $\sum_{i=1}^{4} \frac{1}{3i}$ → $\frac{1}{3(1)}+\frac{1}{3(2)}+\frac{1}{3(3)}+\frac{1}{3(4)}=\frac{119}{180}\to\frac{11}{18}$ — 10) C

A) $\frac{5}{12}$ B) $\frac{1}{12}$ C) $\frac{11}{18}$ D) $\frac{25}{36}$

11) $\sum_{i=1}^{4} 3^i$ → $(3^1)(3^2)(3^3)(3^4)$ → 1120 — 11) D

A) 30 B) 84 C) 39 D) 120

12) $\sum_{i=3}^{5} (i^2 - 9)$ → $(3^2-9)+(4^2-9)+(5^2-9)$ → $0+7+16$ → 23 — 12) A

A) 23 B) -15 C) -3 D) 10

13) $\sum_{k=2}^{4} k(k+2)$ $2(2+2)+3(3+2)+4(4+2)$ → $8+15+24$ → 47 — 13) D

A) 24 B) 32 C) 50 D) 47

14) $\sum_{i=3}^{8} 4$ $(4\times3)+(4\times4)+(4\times5)+(4\times6)+(4\times7)+(4\times8)$ → 132 — 14) C

A) 20 B) 24 C) 132 D) 120

Express the sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

15) 3 + 12 + 27 + . . . + 75 — 15) A

$(3\cdot1^2)\to3$, $(3\cdot2^2)\to12$, $(3\cdot3^2)\to27$, $(3\cdot5^2)\to75$

A) $\sum_{i=1}^{5} 3i^2$ B) $\sum_{i=1}^{5} i^2$ C) $\sum_{i=1}^{5} 3^2 i$ D) $\sum_{i=0}^{5} 3i^2$

16) $\frac{1}{4}+\frac{2}{5}+\frac{1}{2}+\ldots+\frac{4}{5}$ — 16) A

A) $\sum_{i=3}^{12} \frac{i}{i+1}$ $\frac{3}{12}\div3=\frac{1}{4}$ B) $\sum_{i=1}^{n} \frac{i}{i+3}$ C) $\sum_{i=0}^{12} \frac{i}{i+3}$ D) $\sum_{i=1}^{12} \frac{i}{i+3}$

17) 2 + 4 + 6 + . . . + 10 — 17) C

A) $\sum_{i=0}^{5} 2i$ B) $\sum_{i=1}^{5} i^2$ C) $\sum_{i=1}^{5} 2i$ D) $\sum_{i=1}^{5} 2i^2$

$2\times1=2$, $2\times2=4$, $2\times3=6$, $2\times5=10$