HW #25 Examples – Complex Numbers in Polar Form Name_____

Write the complex number in polar form. Express the argument in degrees, rounded to the nearest tenth, if necessary.

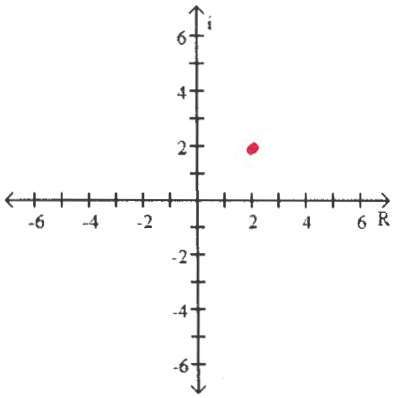
1) $\sqrt{3} - i$ 1) _____

A) $2(\cos 300° + i \sin 300°)$ B) $4(\cos 330° + i \sin 330°)$

C) $4(\cos 300° + i \sin 300°)$ D) $2(\cos 330° + i \sin 330°)$

Plot the complex number in the complex plane.

2) $2 + 2i$ 2) _____



Write the expression in the standard form a + bi.

3) $(\sqrt{3} + i)^5$ 3) _____

A) $9\sqrt{3} + 5i$ B) $-16\sqrt{3} + 16i$ C) $16\sqrt{3} - 16i$ D) $16 - 16\sqrt{3}i$

Find zw or $\frac{z}{w}$ as specified. Leave your answer in polar form.

4) $z = 1 + i$ 4) _____

$w = \sqrt{3} - i$

Find zw.

A) $2\sqrt{2}(\cos 75° + i \sin 75°)$ B) $\frac{\sqrt{2}}{2}(\cos 15° + i \sin 15°)$

C) $2\sqrt{2}(\cos 345° + i \sin 345°)$ D) $2\sqrt{2}(\cos 15° + i \sin 15°)$

5) $z = 1 + i$ 5) _____

$w = 1 - \sqrt{3}i$

Find $\frac{z}{w}$.

A) $\frac{1}{2}(\cos 15° + i \sin 15°)$ B) $\frac{\sqrt{2}}{2}(\cos 105° + i \sin 105°)$

C) $\frac{1}{2}(\cos 105° + i \sin 105°)$ D) $\frac{\sqrt{2}}{2}(\cos 15° + i \sin 15°)$

① $\sqrt{3} - i$ rectangular form

$z = r(\cos\theta + i\sin\theta)$ polar form

$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{Im}{Re}\right)$

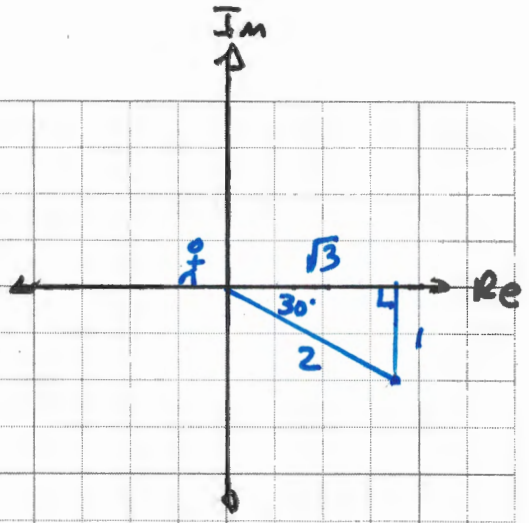
$r = \sqrt{(\sqrt{3})^2+1^2}$ $\quad = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$r = \sqrt{3+1}$ $\quad \theta = 30^\circ$ in Q4

$r = \sqrt{4}$ $\quad \theta = -30$ or $\theta = 330^\circ$

$r = 2$

$$z = 2(\cos 330^\circ + i\sin 330^\circ)$$



③ $(\sqrt{3} + i)^5$ $\quad z^n = r^n[\cos(n\theta) + i\sin(n\theta)]$

$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{Im}{Re}\right)$

$r = \sqrt{(\sqrt{3})^2+1^2}$ $\quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$r = \sqrt{3+1}$ $\quad \theta = 30^\circ$ in Q1

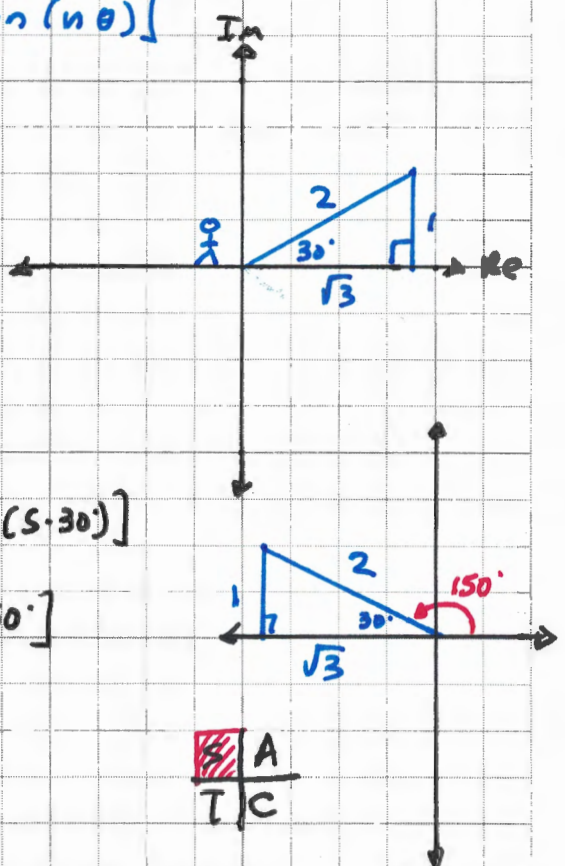
$r = \sqrt{4}$ $\quad \theta = 30^\circ$

$r = 2$ $\quad z^5 = 2^5[\cos(5\cdot 30^\circ) + i\sin(5\cdot 30^\circ)]$

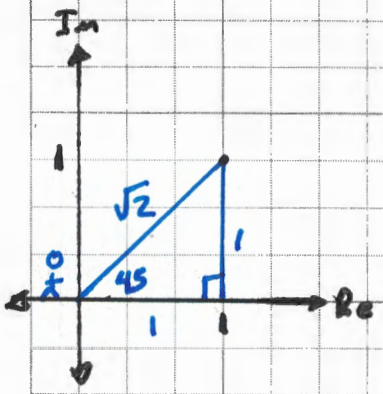
$= 32[\cos 150^\circ + i\sin 150^\circ]$

$= 32\left[-\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right]$

$$z^5 = -16\sqrt{3} + 16i$$



④ $z = 1+i$



$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{Im}{Re}\right)$

$r = \sqrt{1^2+1^2}$ $\quad \theta = \tan^{-1}\left(\frac{1}{1}\right)$

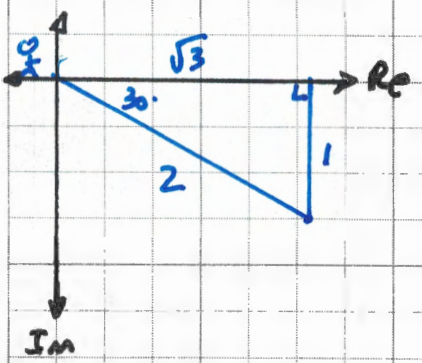
$r = \sqrt{1+1}$ $\quad \theta = \tan^{-1}(1)$

$r = \sqrt{2}$ $\quad \theta = 45°$

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{2}(\cos 45° + i\sin 45°)$

$w = \sqrt{3} - i$



$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{Im}{Re}\right)$

$r = \sqrt{(\sqrt{3})^2+1^2}$ $\quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$r = \sqrt{3+1}$ $\quad \theta = 30°$ in Q4

$r = \sqrt{4}$ $\quad \theta = -30$ or $330°$

$r = 2$

$z = 2(\cos 330° + i\sin 330°)$

$z \cdot w = r_1 \cdot r_2 [\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

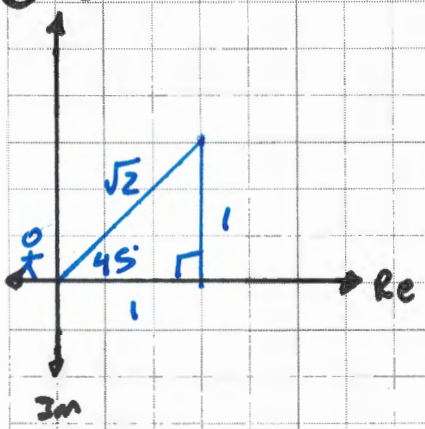
$= \sqrt{2}\cdot 2[\cos(45°+330°) + i\sin(45°+330°)]$

$2\sqrt{2}[\cos(375°) + i\sin(375°)]$

Note: $375° = 15°$

$2\sqrt{2}(\cos 15° + i\sin 15°)$

③ $z = 1+i$



$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{Im}{Re}\right)$

$r = \sqrt{1^2+1^2}$ $\quad \theta = \tan^{-1}\left(\frac{1}{1}\right)$

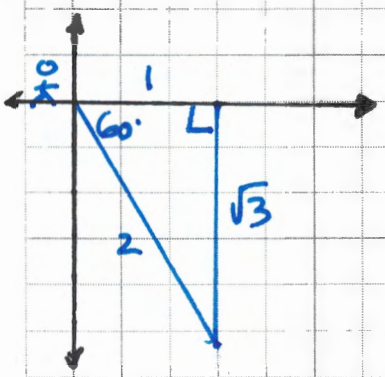
$r = \sqrt{1+1}$ $\quad \theta = \tan^{-1}(1)$

$r = \sqrt{2}$ $\quad \theta = 45°$

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{2}(\cos 45° + i\sin 45°)$

$w = 1 - \sqrt{3}i$



$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{Im}{Re}\right)$

$r = \sqrt{1+(\sqrt{3})^2}$ $\quad \theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$r = \sqrt{1+3}$ $\quad \theta = \tan^{-1}(\sqrt{3})$

$r = \sqrt{4}$ $\quad \theta = 60$ in Q4

$r = 2$ $\quad \theta = -60°$ or $\theta = 300°$

$z = 2(\cos 300° + i\sin 300°)$

$$\frac{z}{w} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

$$\frac{z}{w} = \frac{\sqrt{2}}{2}\left[\cos(45° - 300°) + i\sin(45° - 300°)\right]$$

$$\frac{z}{w} = \frac{\sqrt{2}}{2}\left[\cos(-255°) + i\sin(-255°)\right]$$

$$\frac{z}{w} = \frac{\sqrt{2}}{2}(\cos 105° + i\sin 105°)$$

Note: $-255° = 105°$